

# Effect of flipping noise on the entanglement dynamics of a 3x3 system

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Entanglement dynamics of a qutrit-qutrit system under the influence of global, local and multilocal decoherence introduced by phase flip, trit flip and trit phase flip channels is investigated. The negativity and realignment criterion are used to quantify the entanglement of the system. It is shown that the entanglement sudden death and distillability sudden death can be avoided in the presence of phase flip, trit flip and trit-phase flip environments. It is shown that certain free entangled distillable qutrit states become bound entangled or separable i.e. convert into non-distillable states under different flipping noises. It is also seen that local operations do not have any effect on the entanglement dynamics of the system. Further more, no ESD and DSD is seen for the case of trit flip channel.

Keywords: Quantum channels; qutrit entanglement; global noise.

## I. INTRODUCTION

Quantum entanglement is a fundamental resource for many quantum information processing tasks, e.g. super-dense coding, quantum cryptography and quantum error correction [1-4]. Entangled states can be used in constructing number of protocols, e.g. teleportation [5], key distribution and quantum computation [6]. During recent past, entanglement sudden death (ESD) has been investigated by different authors for bipartite and multipartite states [7-10]. Yu and Eberly [11, 12] have shown that entanglement loss occurs in a finite time under the action of pure vacuum noise in a bipartite qubit system. A geometric interpretation of the phenomenon of ESD has been given in Ref. [13]. Furthermore, experimental evidences of ESD have been reported for optical setups [14, 15] and atomic ensembles [16]. Peres-Horodecki [17, 18] have studied entanglement of qubit-qubit and qubit-qutrit states and established separability criterion. According to this

criterion, the partial transpose of a separable density matrix must have non-negative eigenvalues. For non-separable states, the sum of the absolute values of the negative eigenvalues of the partial transpose matrix gives the degree of entanglement of a density matrix also termed as negativity. Ann et al. [19] have studied a qubit-qutrit system where they have shown the existence of ESD under the influence of dephasing noise.

Bipartite entangled states are divided into free-entangled states and bound-entangled states [20, 21]. Free-entangled states can be distilled under local operations and classical communication (LOCC) whereas bound-entangled states can not be distilled to pure-state entanglement irrespective of the number of copies of the initial state available. Wang et al. proposed that free-entangled states may be converted into bound-entangled states under multi-local [22] and collective dephasing processes [23]. They have shown that certain free-entangled states of qutrit-qutrit systems become non-distillable in a finite time under the influence of classical noise. For a qutrit-qutrit system there are many bound entangled states and no single criterion can fully describe all of them [24]. However, the realignment criterion can be used to detect certain bound entangled states [25].

Since, it is not possible to completely isolate a quantum system from its environment. Therefore, one needs to investigate the behavior of entanglement in the presence of environmental effects. A major problem of quantum communication is to faithfully transmit unknown quantum states through a noisy quantum channel. When quantum information is sent through a channel, the carriers of the information interact with the channel and get entangled with its many degrees of freedom. In order to quantify entanglement, two measures are the negativity [26], a measure of a state having negative partial transpose and the realignment criterion [25]. The negativity is equal to the absolute value of the sum of negative eigenvalues of partial transpose of a state. When the negativity becomes zero, we need to study the time evolution of realignment criterion to determine the existence of bound-entangled states. However, it should be kept in mind that realignment criterion could not detect all entangled states.

In this paper, the effect of flipping noise on the entanglement dynamics of a qutrit-qutrit system under the influence of global, local and multilocal decoherence is investigated by considering phase flip, trit flip and trit phase flip channels. It is shown that the entanglement sudden death can be avoided in the presence of phase flip, trit flip and trit phase flip environments. It is also shown that particular local operations cannot avoid the non-distillability of the distillable states. Furthermore, no ESD and DSD is seen for trit flip channel in the presence of global noise.

## II. DYNAMICS OF A QUTRIT-QUTRIT SYSTEM UNDER FLIPPING NOISE

Let us consider that the initial state is interacting with the noisy environment both collectively and individually. Local and multi-local couplings describe the situation when both the qutrits are independently influenced by their individual noisy environments. Whereas, the global decoherence corresponds to the situation when it is influenced by both local and multilocal noises at the same time. The state shared by the two parties is an entangled qutrit-qutrit state of the form

$$\rho(0) = \frac{2}{7} \left( (|\Psi_+\rangle \langle \Psi_+| + \frac{\alpha}{7} \sigma_+ + \frac{5-\alpha}{7} \sigma_-) \right) \quad (1)$$

where  $2 \leq \alpha \leq 5$ ,  $|\Psi_+\rangle = \frac{1}{\sqrt{3}}(|00\rangle + |11\rangle + |22\rangle)$  is a maximally entangled bipartite qutrit state and  $\sigma_{\pm}$  are separable states given as  $\sigma_+ = 1/3(|01\rangle \langle 01| + |12\rangle \langle 12| + |20\rangle \langle 20|)$ ,  $\sigma_- = 1/3(|01\rangle \langle 01| + |21\rangle \langle 21| + |02\rangle \langle 02|)$

The above density matrix is separable for  $2 \leq \alpha \leq 3$ , bound entangled for  $3 \leq \alpha \leq 4$  and free entangled for  $4 < \alpha \leq 5$  [27]. The interaction between the system and its environment introduces the decoherence to the system, which is a process of the undesired correlation between the system and the environment. The dynamics of the composite system in the presence of flipping noise can be described in Kraus operators formalism [29]

$$\rho_f = \sum_k E_k \rho_i E_k^\dagger, \quad (2)$$

where the Kraus operators  $E_i$  satisfy the following completeness relation

$$\sum_k E_k^\dagger E_k = I. \quad (3)$$

The single qutrit Kraus operators for phase flip channel can be written as

$$E_0 = \sqrt{1 - \frac{2p}{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad E_1 = \sqrt{\frac{p}{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-\frac{2\pi i}{3}} & 0 \\ 0 & 0 & e^{\frac{2\pi i}{3}} \end{pmatrix}, \quad E_2 = \sqrt{\frac{p}{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{\frac{2\pi i}{3}} & 0 \\ 0 & 0 & e^{-\frac{2\pi i}{3}} \end{pmatrix}, \quad (4)$$

and the single qutrit Kraus operators for the trit flip channel are given as

$$E_0 = \sqrt{1 - \frac{2p}{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad E_1 = \sqrt{\frac{p}{3}} \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad E_2 = \sqrt{\frac{p}{3}} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad (5)$$

Whereas, the single qutrit Kraus operators for the trit phase flip channel are given by

$$E_0 = \sqrt{1 - \frac{2p}{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad E_1 = \sqrt{\frac{p}{3}} \begin{pmatrix} 0 & 0 & e^{\frac{2\pi i}{3}} \\ 1 & 0 & 0 \\ 0 & e^{-\frac{2\pi i}{3}} & 0 \end{pmatrix},$$

$$E_2 = \sqrt{\frac{p}{3}} \begin{pmatrix} 0 & e^{\frac{-2\pi i}{3}} & 0 \\ 0 & 0 & e^{\frac{2\pi i}{3}} \\ 1 & 0 & 0 \end{pmatrix}, \quad E_3 = \sqrt{\frac{p}{3}} \begin{pmatrix} 0 & e^{\frac{2\pi i}{3}} & 0 \\ 0 & 0 & e^{-\frac{2\pi i}{3}} \\ 1 & 0 & 0 \end{pmatrix}$$

where  $p = 1 - e^{-\Gamma t}$  represents the quantum noise parameter usually termed as decoherence parameter. Here the bounds  $[0, 1]$  of  $p$  correspond to  $t = 0, \infty$  respectively. The evolution of the initial density matrix of the composite system when it is influenced by local and multi-local environments is given in Kraus operator form as

$$\rho_f = \sum_{i,j,k} (E_j^B E_k^A) \rho_{AR} (E_j^B E_k^A)^\dagger \quad (6)$$

and the evolution of the system when it is influenced by global environment is given in Kraus operator representation as

$$\rho_f = \sum_{i,j,k} (E_i^{AB} E_j^B E_k^A) \rho_{AR} (E_i^{AB} E_j^B E_k^A)^\dagger \quad (7)$$

where  $E_k^A = E_m^A \otimes I_3$ ,  $I_2 \otimes E_j^B$  are the Kraus operators of the multilocal couplings of the individual qutrits and  $E_i^{AB} = E_m^A \otimes E_n^A$  are the Kraus operators of the collective coupling of the qutrit system. Using equations (2-5) along with the initial density matrix of as given in equation (1) and taking the partial transpose over the second qutrit, the eigenvalues of the final density matrix can be easily found. Let the decoherence parameters for local and global noise be  $p_1$ ,  $p_2$  and  $p$  respectively. The eigenvalues of the partial transpose matrix, when both qutrits are influenced by multi-local noise of the phase flip channel, are given as

$$\lambda_1 = \lambda_2 = \lambda_3 = \frac{2}{21}$$

$$\lambda_{4,5,6} = \frac{1}{42} \left[ 5 - \sqrt{\frac{4\alpha^2 - 20\alpha + 16p_1^2(p_2 - 1)^2}{-32p_1(p_2 - 1)^2 + 16p_2^2 - 32p_2 + 41}} \right] \quad (8)$$

$$\lambda_{7,8,9} = \frac{1}{42} \left[ 5 + \sqrt{\frac{4\alpha^2 - 20\alpha + 16p_1^2(p_2 - 1)^2}{-32p_1(p_2 - 1)^2 + 16p_2^2 - 32p_2 + 41}} \right] \quad (9)$$

and the realignment criterion in this case reads

$$||\rho^R|| - 1 = \frac{2}{21}\sqrt{3A_1^2 - 15A_1 + 19} + \frac{4}{7}\sqrt{(p_2p_1 - p_1 - p_2 + 1)^2} - \frac{26}{3} \quad (10)$$

The expressions for local noise can be obtained by setting  $p_2 = 0$ . The eigenvalues of the partial transpose matrix, when both the qutrits are influenced by the global noise of phase flip channel, are given by

$$\lambda_1 = \lambda_2 = \lambda_3 = \frac{2}{21} \quad (11)$$

$$\lambda_{4,5,6} = \frac{1}{42} \left( \sqrt{\begin{matrix} 16p^8 - 128p^7 + 448p^6 - 896p^5 + 1120p^4 \\ -896p^3 + 448p^2 - 128p + 4\alpha^2 - 20\alpha + 41 \end{matrix}} + 5 \right) \quad (12)$$

$$\lambda_{7,8,9} = \frac{1}{42} \left( \sqrt{\begin{matrix} 16p^8 - 128p^7 + 448p^6 - 896p^5 + 1120p^4 \\ -896p^3 + 448p^2 - 128p + 4\alpha^2 - 20\alpha + 41 \end{matrix}} - 5 \right) \quad (13)$$

and the realignment criterion in this case reads

$$||\rho^R|| - 1 = \frac{4p^4}{7} - \frac{16p^3}{7} + \frac{24p^2}{7} - \frac{16p}{7} + \frac{2}{21}\sqrt{3\alpha^2 - 15\alpha + 19} - \frac{2}{21} \quad (14)$$

The eigenvalues of the partial transpose matrix when both the qutrits are influenced by the trit flip and trit phase flip channels are not provided with their analytical relations as these are too lengthy in size. Therefore, they are just interpreted in the graphs. The entanglement for all mixed states  $\rho_{AB}$  can be quantified by the negativity

$$N(\rho_{AB}) = \max\{0, \sum_k |\lambda_k^{T_A(-)}|\} \quad (15)$$

where  $\lambda_k^{T_A(-)}$  represents the negative eigenvalues of the partial transpose of the density matrix  $\rho_{AB}$ .

### III. DISCUSSIONS

In this work, the effect of decoherence on a qutrit-qutrit system is investigated. In figure 1, the negativity and realignment criterion are plotted as a function of decoherence parameter  $p$  for local noise introduced through phase flip, trit flip and trit phase flip channels. It is seen that the partial transpose criterion fails to detect the behavior of entanglement in density matrices of larger  $\alpha$  for higher values of decoherence. The quantity  $||\rho^R|| - 1$  is positive for  $p < 0.5$  which shows that these density matrices are bound entangled. However for  $p > 0.5$ , the realignment criterion also fails to detect the possible entanglement for these states.

In figure 2, negativity and realignment criterion are plotted as a function of decoherence parameter  $p$  for multi-local noise introduced through phase flip, trit flip and trit phase flip channels. It can be seen that no ESD occurs in any density matrix as the negativity is positive for all the density matrices. However, for  $p < 0.1$ , some states that correspond to large values of  $\alpha$  become positive partial transpose states (PPT).

In figure 3, negativity and realignment criterion are plotted as a function of decoherence parameter  $p$  for global noise introduced through phase flip, trit flip and trit phase flip channels. It is seen that the free entangled distillable states convert into bound entangled or separable states and therefore become completely non-distillable in the presence of global noise. In figure 4, three-dimensional graphs for negativity are plotted as a function of decoherence parameter  $p$  and parameter  $\alpha$  for multi-local and global noises introduced through phase flip, trit flip and trit phase flip channels. It is seen that ESD and DSD can be completely avoided for the case of trit flip channel.

In order to see the effect of local unitary operation on the dynamics of qutrit-qutrit state, let the unitary operator  $U = I_3 \otimes \theta$ , with  $\theta = |0\rangle\langle 1| + |1\rangle\langle 0| + |2\rangle\langle 2|$ . By applying this operator locally to the state (equation 1), it converts the maximally entangled state  $|\Psi_+\rangle$  into another maximally entangled state given by  $|\tilde{\Psi}_+\rangle = \frac{1}{\sqrt{3}}(|01\rangle + |10\rangle + |22\rangle)$  and the two separable states  $\sigma_{\pm}$  are transformed to  $\tilde{\sigma}_+ = 1/3(|00\rangle\langle 00| + |12\rangle\langle 12| + |21\rangle\langle 21|)$ ,  $\tilde{\sigma}_- = 1/3(|11\rangle\langle 11| + |20\rangle\langle 20| + |02\rangle\langle 02|)$ , the final density matrix takes the form

$$\sigma_{\alpha} = U\rho_{\alpha}U^{\dagger} = \frac{2}{7} \left( (|\tilde{\Psi}_+\rangle\langle \tilde{\Psi}_+| + \frac{\alpha}{7}\tilde{\sigma}_+ + \frac{5-\alpha}{7}\tilde{\sigma}_- \right) \quad (16)$$

The evolution of  $\sigma_{\alpha}$  in the presence of flipping noise and its partial transpose can be found in a similar fashion as calculated for equation (5). It is seen that the eigenvalues for each case remains unchanged. Therefore, the local operation does not change the behavior of entanglement in the presence of flipping noise.

#### IV. CONCLUSIONS

Entanglement dynamics of a qutrit-qutrit system under the influence of global, local and multilocal flipping noise is investigated. In order to quantify the entanglement of the system, the negativity and realignment criterion are used. It is shown that the entanglement sudden death and distillability sudden death can be avoided in the presence of phase flip, trit flip and trit phase flip environments. It is shown that certain free entangled distillable qutrit states become bound entangled or separable i.e. convert into non-distillable states under different flipping noises. It is

also seen that local operations do not have any effect on the entanglement dynamics of the system. Further more, no ESD and DSD is seen for the case of trit flip channel.

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## Figures captions

**Figure 1.** (Color online). The negativity and realignment criterion are plotted as a function of decoherence parameter  $p$  for local noise introduced through phase flip, trit flip and trit phase flip channels.

**Figure 2.** (Color online). The negativity and realignment criterion are plotted as a function of decoherence parameter  $p$  for multi-local noise introduced through phase flip, trit flip and trit phase flip channels.

**Figure 3.** (Color online). The negativity and realignment criterion are plotted as a function of decoherence parameter  $p$  for global noise introduced through phase flip, trit flip and trit phase flip channels.

**Figure 4.** (Color online). The negativity is plotted as a function of decoherence parameter  $p$  and parameter  $\alpha$  for multi-local and global noises introduced through phase flip, trit flip and trit phase flip channels.

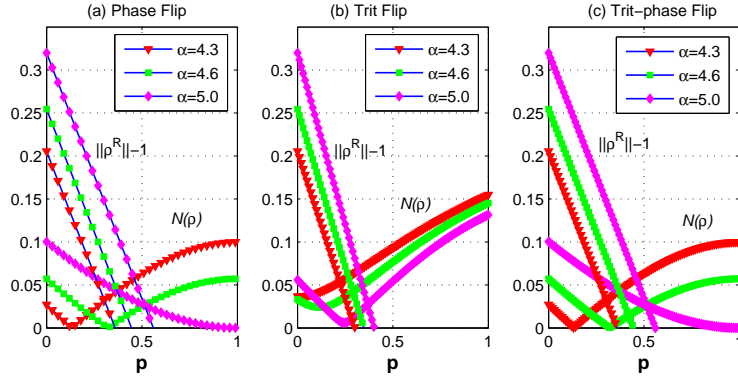


FIG. 1: (Color parameter  $p$  for

of decoherence  
als.

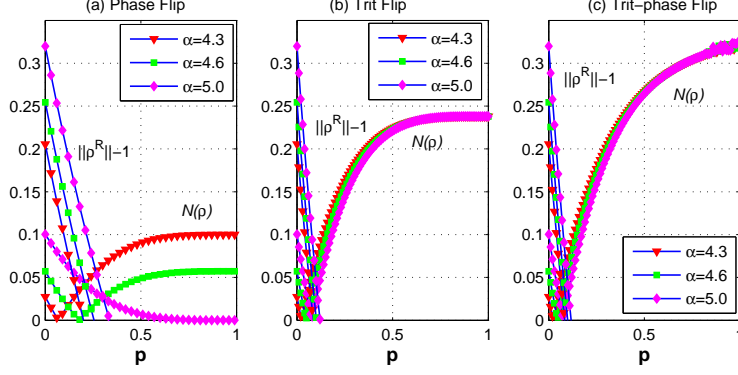


FIG. 2: (Color online). The negativity and realignment criterion are plotted as a function of decoherence parameter  $p$  for multi-level noise introduced through phase flip, trit flip and trit phase flip channels.

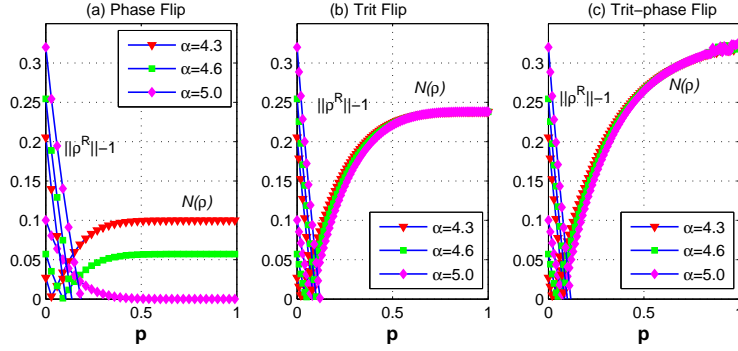


FIG. 3: (Color online). The negativity and realignment criterion are plotted as a function of decoherence parameter  $p$  for global noise introduced through phase flip, trit flip and trit phase flip channels.

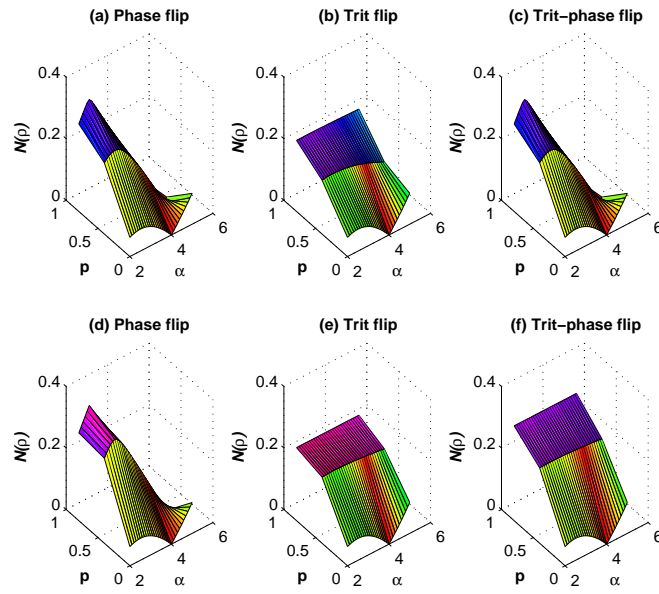


FIG. 4: (Color online). The negativity is plotted as a function of decoherence parameter  $p$  and parameter  $\alpha$  for multi-local and global noises introduced through phase flip, trit flip and trit phase flip channels.